Determination of the Principal Directions of Composite Helicopter Rotor Blades with Arbitrary Cross Sections

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Modern helicopter rotor blades with non-homogeneous cross sections, composed of anisotropic material, require highly sophisticated structural analysis because of various cross sectional geometry and material properties. They may be subjected by the combined axial, bending, and torsional loading, and the dynamic and static behaviors of rotor blades are seriously influenced by the structural coupling under rotating condition. To simplify the analysis procedure using one dimensional beam model, it is necessary to determine the principal coordinate of the rotor blade. In this study, a method for the determination of the principal coordinate including elastic and shear centers is presented, based upon continuum mechanics. The scheme is verified by comparing the results with confirmed experimental results.

Key Words : Rotor Blade, Cross Sectional Property, Principal Direction, Helicopter Blade

1. Introduction

Different from the fixed wings of aircraft, helicopter rotor blades play an important role in control as well as in thrust and lift, and the technique to minimize the vibration, caused by structural mass and repeated loads, is indispensable to analyse and design the helicopter rotor blades. Composite materials are good choice of the blade materials in that they have excellent stiffness, durability, and manufacturability of the monolithic construction (Pinkey, 1974). The composite blades have simplified hubs, instead of conventional hubs which are composed of three hinges to receive forces in three directions without reaction.

Simplified hubs have advantages in maintenance and reliability, but have disadvantages in complicated cross sections and anisotropicity (Joo, 1993). Most of structural models, developed in the past, were analysed using beams with simple cross section such as circular tube, I-beam, rectangular box, and so on.

Three dimensional finite elements can be employed, but this "brute force" technique, as pointed out by Hodges (1990), is quite expensive, and the form of the results is not amendable to easy interpretation. Since rotor blades are of long shape, one dimensional model may be feasible at least from the view point of computation. However, although one dimensional or beam kinematics can be formulated in a rather elegant fashion, constitutive laws in terms of three dimensional elastic constants for small strains can only give approximations when the structure is to be treated as one dimensional. In this case, it is a key to minimize the latent error caused by one dimensional modelling. (Jung, 1993)

The modelling of composite rotor blades has been made in various ways, but they can be arranged in two classes, remarkably. In the first method, cross sectional properties such as shear center, warping function and rigidity are obtained through two dimensional linear analysis, con-

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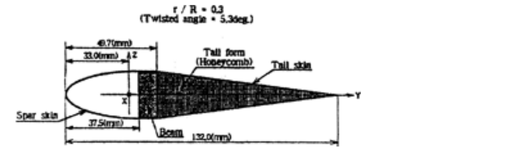
sidering the anisotropy of composite materials (Wondle, 1982; Giavotto et. al., 1983; Jung and Kim, 1996). The second method derives governing equation from kinematics considering geometric nonlinearity, cross sectional warping, shear effects, and so on (Bauchau and Hong, 1987; Chandra and Chopra 1992; Hodges, 1979; Yoo et. al., 1997). But the former cannot describe the cross sectional geometry sufficiently, because it assumes the rotor blade to be simple box cantilevered beam. In the same time, it is a heavy burden to analyze the two dimensional cross section early in the design of rotor blade. While the other have an application trouble, because the governing equation has to be derived each time whenever configuration of the rotor blade is changed.

Many efforts are attempted to propose a brief modelling technique. Among their efforts, there is a method which has been studied using equivalent stiffness matrix of rotor blade (Rapp and Wondle, 1992; Yu et. al., 1995; Yu et. al., 1996). However, in this case, it is a key for accurate solution to treat coupling terms. To deal with coupling terms properly, it is necessary to determine the principal coordinate including elastic and shear centers. Also principal directions are significant in the design of rotor blade geometry.

In this study, the principal coordinates are determined using equivalent stiffness matrix, based upon the concepts of equivalent energy and matrix operation. To verify this study, the scheme is applied to the models of confirmed known results.

2. Equivalent Stiffness Matrix

Rotor hubs used to have three hinges to remove the stress action on the root of rotor blade, since there had been no material which can support against flapping, lag, and feathering, simultaneously. Three hinges make the structure of hub complicate mechanically and have difficulties in maintenance and performance. Rotor blades can be substituted to the type of flexible beam by developing composite materials which can control the required directional rigidity by choosing



R : Total length of the blade
r : Longitudinal position of the blade
r / R = 0 : Root, r / R = 0.5 : Mid position, r / R = 1 : End

Fig. 1 Illustration of blade section

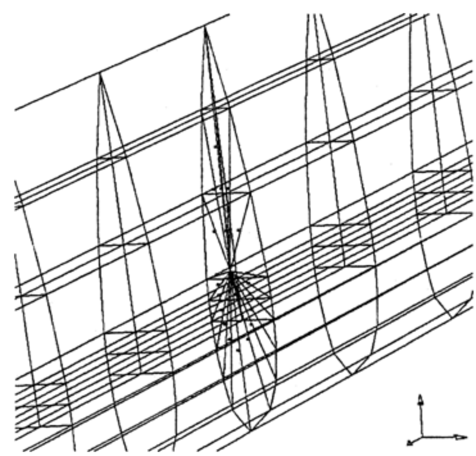


Fig. 2 Illustration of rigid modelling

the ply angles and the order of composite laminates like fiber reinforced plastics, but the structural analysis of the blades become difficult relatively.

Rotor blades may be subjected to combined axial, bending, and torsional loading. The dynamic and static behaviors of rotor blades are seriously influenced by structural coupling under rotating condition. Therefore it is very important to determine the principal coordinates of the structure of rotor blades to treat the complexity of coupling terms in the governing equations of motion.

The rotor blade introduced in this study, as shown in Fig. 1, has no hinge. The inner part of rotor blade is made of wood and foam. The wood gives the rotor blade rigidity but the foam makes only shape of the blade and it is attached to the tail. The whole part is coated by composite material. The twisted angle from fixed end to free end is 15 degree.

To extract equivalent stiffness of a cross section of the blade, an arbitrary node is given to the cross section of blade, as shown in Fig. 2, the node has no physical meaning. The equivalent stiffness matrix is extracted from the given node using the concepts of reduction in the finite element method. From the equivalent stiffness matrix, the principal coordinate system can be obtained by using beam theory which includes shear center, elastic center, and principal directions.

3. Principal Coordinate System

Arbitrary cross section of cantilevered beam, as shown in Fig. 3, has two origins O and O′ whose distance is a in y-direction, b in z-direction from origin O. Displacement vector can be expressed as below at origin O.

$$\{u\}^T=(x\ y\ z\ \theta_x\ \theta_y\ \theta_z)$$

Assume that the coordinate systems expressed by origin O, O′ and displacement $\{u\}$, $\{u'\}$ are in principal direction and displacement vector $\{u'\}$ can be expressed as below at origin O′ which is the shear center

$$\{u\}^T=(x'\ y'\ z'\ \theta_x'\ \theta_y'\ \theta_z')$$

The relationship between load vector $\{f\}$ and displacement vectors $\{u\}$ and $\{u'\}$ is as below

$$\{f\}=[A]\{u\}=[B]\{u'\}$$

where $[A]$ and $[B]$ are stiffness matrices of 6 ×6.

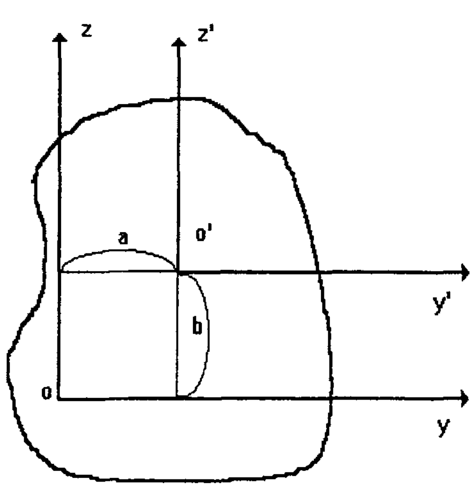


Fig. 3 Illustration of coordinate system

The relation between $\{u\}$ and $\{u'\}$ can be expressed as below

$$\begin{aligned} x'&=x+b\theta_y-a\theta_z && (a)\\ y'&=y-b\theta_x && (b)\\ z'&=z+a\theta_x && (c)\\ \theta_x'&=\theta_x && (d)\\ \theta_y'&=\theta_y && (e)\\ \theta_z'&=\theta_z && (f) \end{aligned} \tag{1}$$

where a and b represent constants related to the material.

Stiffness matrix $[A]$ is of shape as below, because they are in principal direction, as in Ref. (Kardestuncer and Norrie, 1987).

$$\begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & 0\\ 0 & a_{22} & 0 & 0 & 0 & a_{26}\\ 0 & 0 & a_{33} & 0 & a_{35} & 0\\ 0 & 0 & 0 & a_{44} & 0 & 0\\ 0 & 0 & a_{53} & 0 & a_{55} & 0\\ 0 & a_{62} & 0 & 0 & 0 & a_{66} \end{bmatrix} \tag{2}$$

where a_{ij}'s are constants, $i, j=1, 2, 3, 4, 5, 6$

Force-displacement relationship can be divided into two categories. First, to determine the elastic center, the equations related to the elastic center of force and displacement can be written, as in Ref. (Rapp and Wondle, 1992).

$$\begin{aligned} \{f_e\}&=[A_e]\{u_e\}=[B_e]\{u'_e\}\\ &=\begin{bmatrix} a_{11} & 0 & 0\\ 0 & a_{55} & 0\\ 0 & 0 & a_{66} \end{bmatrix}\begin{Bmatrix} x\\ \theta_y\\ \theta_z \end{Bmatrix}\\ &=\begin{bmatrix} b_{11} & b_{15} & b_{16}\\ b_{51} & b_{55} & b_{56}\\ b_{61} & b_{65} & b_{66} \end{bmatrix}\begin{Bmatrix} x'\\ \theta_y'\\ \theta_z' \end{Bmatrix} \end{aligned} \tag{3}$$

where a_{ij}'s are constant, $i, j=1, 2, 3, 4, 5, 6$

From the concepts of the energy conservation,

$$\frac{1}{2}\{u_e\}^T[A_e]\{u_e\}=\frac{1}{2}\{u'_e\}^T[B_e]\{u'_e\} \tag{4}$$

The subscript e means that the above terms are with respect to the elastic center.

Expanding the above, and a and b become

$$a=\frac{b_{16}}{b_{11}},\ b=-\frac{b_{15}}{b_{11}} \tag{5}$$

To determine the shear center, the equations related to the shear center of force and displacement are written as below (Rapp and Wondle, 1992).

$$
\begin{aligned}
\{f_s\} &= [A_s]\{u_s\} = [B_s]\{u'_s\} \\
&= \begin{bmatrix} a_{22} & 0 & 0 \\ 0 & a_{33} & 0 \\ 0 & 0 & a_{44} \end{bmatrix} \begin{Bmatrix} y \\ z \\ \theta_x \end{Bmatrix} \\
&= \begin{bmatrix} b_{22} & b_{23} & b_{24} \\ b_{23} & b_{33} & b_{34} \\ b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{Bmatrix} y' \\ z' \\ \theta'_z \end{Bmatrix}
\end{aligned}
\tag{6}
$$

In the same way, a and b can be expressed by

$$a = -\frac{b_{34}}{b_{33}},\ b = \frac{b_{24}}{b_{22}} \tag{7}$$

Secondly, to determine principal directions, rotational transformation is used and expressed by

$$
[R] = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & \cos\theta & -\sin\theta & 0 & 0 & 0 \\
0 & \sin\theta & \cos\theta & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & \cos\theta & -\sin\theta \\
0 & 0 & 0 & 0 & \sin\theta & \cos\theta
\end{bmatrix} \tag{8}
$$

and, relation between rotated stiffness matrix by θ, $[K_g]$, and stiffness matrix in principal coordinates, $[K_p]$, is

$$[K_p] = [R]^T[K_g][R] \tag{9}$$

From the above, the principal direction θ can be found in Eq. (10).

$$
\begin{aligned}
\theta &= \frac{1}{2}\tan^{-1}\left(\frac{2K_{g56}}{K_{G66} - K_{g55}}\right) \\
&= \frac{1}{2}\tan^{-1}\left(\frac{2K_{g23}}{K_{g33} - K_{g22}}\right)
\end{aligned}
\tag{10}
$$

4. Results and Discussion

To verify the orthogonal transformation, a cantilevered beam of which cross section is regular tetragon is introduced. The stiffness matrix, which is rotated by 30° in three dimensional finite element model, is compared with the stiffness matrix which is calculated by Eq. (6)

$$\{f\} = [K]\{u\} \tag{11}$$

where

$$\{f\}^T = \{F_x, F_y, F_z, M_x, M_y, M_z\}$$

$$\{u\}^T = \{x, y, z, \theta_x, \theta_y, \theta_z\}$$

The stiffness matrix by Eq. (6) is as below in Eq. (12)

$$
\begin{bmatrix}
2.3706e{+}004 & 0 & 0 & 0 & 0 & 0 \\
0 & 6.0150e{+}003 & 0 & 0 & 0 & -3.0075e{+}004 \\
0 & 0 & 6.0150e{+}003 & 0 & 3.0075e{+}004 & 0 \\
0 & 0 & 0 & 1.6040e{+}004 & 0 & 0 \\
0 & 0 & 3.0075e{+}004 & 0 & 1.8007e{+}005 & 0 \\
0 & -3.0075e{+}004 & 0 & 0 & 0 & 1.8007e{+}005
\end{bmatrix} \tag{12}
$$

and the stiffness matrix by finite element model is as below in Eq. (13)

$$
\begin{bmatrix}
2.3706e{+}004 & 0 & 0 & 0 & 0 & 0 \\
0 & 6.0150e{+}003 & 0 & 0 & 0 & -3.0075e{+}004 \\
0 & 0 & 6.0150e{+}003 & 0 & 3.0075e{+}004 & 0 \\
0 & 0 & 0 & 1.6040e{+}004 & 0 & 0 \\
0 & 0 & 3.0075e{+}004 & 0 & 1.8007e{+}005 & 0 \\
0 & -3.0075e{+}004 & 0 & 0 & 0 & 1.8007e{+}005
\end{bmatrix} \tag{13}
$$

Two stiffness matrix agree with each other very well.

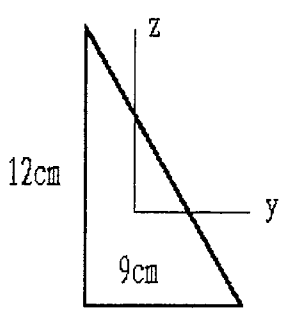


Fig. 4 Cross section of right-angled triangle cantilever beam

Next, for a cantilevered beam whose cross section is a right triangle having a height of 120mm and a width of 90mm as shown in Fig. 4, equivalent stiffness matrix at free end is as in Eq. (14) in the principal direction.

$$\begin{bmatrix} 6.0076e+004 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.7165e+004 & 0 & 0 & 0 & -1.0385e+005 \\ 0 & 0 & 1.7165e+004 & 0 & 1.0385e+005 & 0 \\ 0 & 0 & 0 & 2.0952e+005 & 0 & 0 \\ 0 & 0 & 1.0385e+005 & 0 & 1.2135e+006 & 0 \\ 0 & -1.0385e+005 & 0 & 0 & 0 & 8.3452e+005 \end{bmatrix} \tag{14}$$

As mentioned above terms of (2, 3), (3, 2), (5, 6), (6, 5), (2, 5), (3, 6), (5, 2), (6, 3) in the equivalent stiffness matrix have to be eliminated in principal coordinate system and the above results meet this requirements very well.

When this right triangular cantilevered beam get twisted by 15 degrees from fixed end to free end, the comparison of twisted angle and principal direction with respect to the longitudinal position of the beam is shown in Fig. 5. It is natural for principal direction to have a same proportional tendency with twisted angle because of homogeneous material.

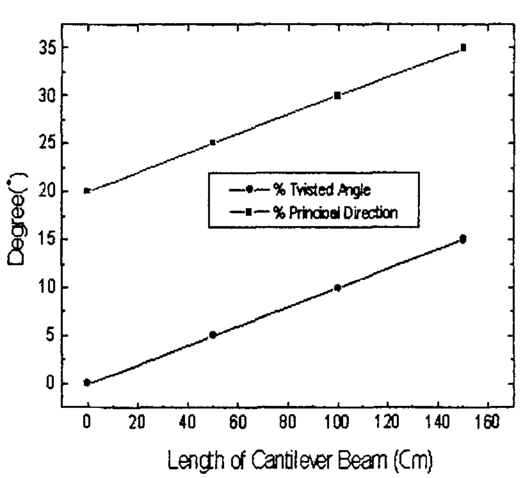


Fig. 5 Principal direction of right-angled triangle cantilever beam

On the basis of the above mentioned, The comparison of twisted angle and principal direction of the rotor blade whose cross section is as explained in Fig. 1 is shown in Fig. 6 where two lines are not parallel. The reason is that the rotor blade material is not homogeneous, but is laminated composite, of which ply angle accumlation order are designed differently along the longitudinal direction to assure the reguired rigidity of the simplified hub portion. The locations of the elastic and the shear centers, were determined experimentally in Ref. (Joo, 1993), in which real blade model NACA-0012 was adapted as shown in Fig. 7.

The rotor blade is made up of spar, tail skin, weight balance, and tail foam which are built in glass fiber roving, fabric woven, lead, and poly urethan, respectively

The shape of the rotor blade of NACA-0012 has an axis of symmetry, since one of the principal directions, is along the axis of symmetry. In

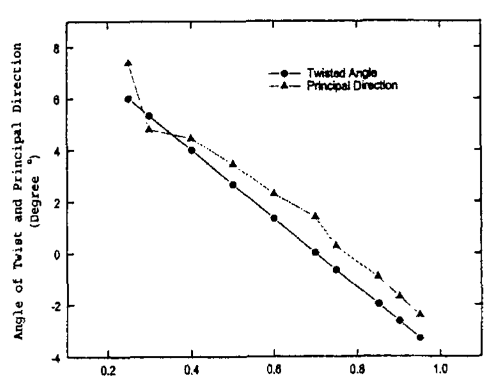


Fig. 6 Principal direction of composite rotor blade

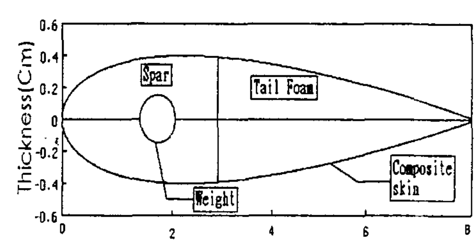


Fig. 7 Illustration of reference blade

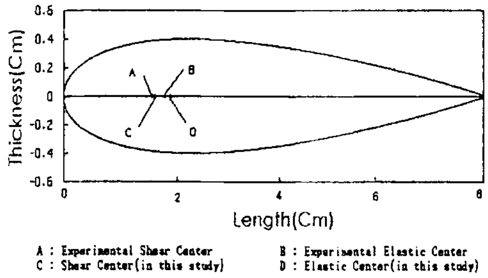


Fig. 8 Center of reference blade

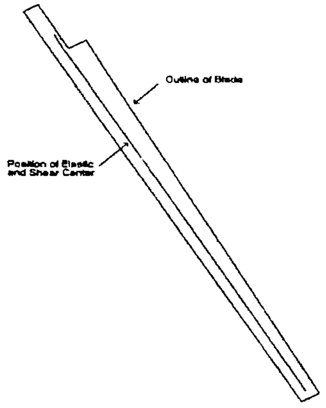


Fig. 9 Position of elastic and shear center

Fig. 8 the locations of the elastic and the shear centers obtained using the scheme of this study are represented and compared with the centers found by the experiments in Ref. (Joo, 1993).

Two locations of the elastic and shear centers, agree well. Since the parts of fixed end of the blade may not be fully constraint in Ref. (Joo, 1993), it is considered that the minute discrepancies are caused by constraint condition.

The shear center is defined to be that the point, given any cross-sectional configuration, at which a shear load produces no twisting, and can be determined by intersecting the resultants of shear force in the principal direction. But it is very difficult problem to use these definitions in practice. Therefore it has been paid dear to determinate the shear center as in Ref. (Wondle, 1982), in which a special finite element has been developed. The method using equivalent stiffness matrix in this paper has something to benefit not only in saving efforts but also in applications, because principal coordinate including elastic and shear centers can be readily obtained using Eqs. (5), (7) and (10) after the components of the equivalent stiffness matrix are evaluated. In Fig. 9 the locations of the elastic and the shear centers are indicated for the rotor blade whose cross section is as explained in Fig. 1. They are almost coincided, and they look like one line.

5. Conclusion

Modern helicopter rotor blades with non-homogeneous cross section which are made from anisotropic material require highly sophisticated structural analysis. Variation in cross section geometry makes the task of analysis more complicated. Because they may be subjected to combined axial, bending, and torsional loading, the dynamic and static behavior of rotor blades is seriously influenced by structural coupling under rotating conditions, which depend upon the principal coordinate system. To determine the principal coordinate system, equivalent stiffness matrix is derived by reduction technique in the finite element method, and the principal coordinate system is determined using simple matrix operations with the elastic and shear centers obtained using the equivalent stiffness matrix. This method is also applied to the model of experiments in Ref. (Joo, 1993). The results are in good accordance with each other.

The scheme to determine the principal coordinate system in this study is quite simple and accurate compared with other methods, since only matrix rotation transformation is used to get uncoupled principal coordinates with equivalent stiffness matrix. This can be utilized in the design of helicopter rotor blades, with saved effort and time.

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